OCR
RECOGNISING ACHIEVEMENT

Friday 1 June 2012 – Morning

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex numbers z and w are given by $z = 6 - \mathrm{i}$ and $w = 5 + 4\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $z + 3w$, **[2]**

(ii) $\dfrac{z}{w}$. **[3]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find

(i) **AB**, **[2]**

(ii) $\mathbf{B}^{-1}\mathbf{A}^{-1}$. **[3]**

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3\mathrm{i}$. Find the values of a and b. **[4]**

4 Find $\displaystyle\sum_{r=1}^{n} (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. **[7]**

5 Prove by induction that, for $n \geqslant 1$, $\displaystyle\sum_{r=1}^{n} 4\times 3^r = 6(3^n - 1)$. **[5]**

6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β.

(i) Use the substitution $x = \dfrac{1}{u+1}$ to obtain a quadratic equation in u with integer coefficients. **[3]**

(ii) Hence, or otherwise, find the value of $\left(\dfrac{1}{\alpha} - 1\right)\left(\dfrac{1}{\beta} - 1\right)$. **[3]**

7 The loci C_1 and C_2 are given by $|z - 3 - 4\mathrm{i}| = 4$ and $|z| = |z - 8\mathrm{i}|$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[6]**

(ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4\mathrm{i}| \leqslant 4 \text{ and } |z| \geqslant |z - 8\mathrm{i}|.$$ **[2]**

8 **(i)** Show that $\dfrac{1}{r} - \dfrac{1}{r+2} \equiv \dfrac{2}{r(r+2)}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=1}^{n} \frac{2}{r(r+2)}$. **[6]**

(iii) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N. **[4]**

9 **(i)** The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **X**. **[2]**

(ii) The matrix **Z** is given by $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2+\sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1-2\sqrt{3}) \end{pmatrix}$. The transformation represented by **Z** is equivalent to the transformation represented by **X**, followed by another transformation represented by the matrix **Y**. Find **Y**. **[5]**

(iii) Describe fully the geometrical transformation represented by **Y**. **[2]**

10 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

(i) Find the determinant of **D** in terms of a. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + 2y - z &= 0 \\ 2x + ay + z &= a \\ x + y + az &= a \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 2$

(c) $a = 0$ **[7]**

Question			Answer	Marks	Guidance	
1	(i)		21 +11i	B1 B1 **[2]**	Real part correct Imaginary part correct	
1	(ii)		 26 – 29i $\frac{26}{41}-\frac{29}{41}\mathrm{i}$	M1 A1 A1 **[3]**	Multiply by conjugate of denominator or find a pair of simultaneous equations Obtain correct numerator or real part Obtain correct denominator or imaginary part	
2	(i)		$\begin{pmatrix}5 & 2\\13 & 6\end{pmatrix}$	M1 A1 **[2]**	Multiplication attempt, 2 elements correct All elements correct	
2	(ii)		EITHER $\mathbf{B}^{-1}\mathbf{A}^{-1}=(\mathbf{AB})^{-1}$ $\frac{1}{4}\begin{pmatrix}6 & -2\\-13 & 5\end{pmatrix}$ OR	 B1 B1ft B1ft **[3]** B1 B1 B1	 Stated or used Divide by correct determinant Both diagonals correct Either inverse correct Two elements correct in final answer , both inverses must be correct All elements correct	

Question			Answer	Marks	Guidance	
3			EITHER			
				M1	Use sum of root and conjugate	
			$a = -8$	A1	Obtain correct answer	
				M1	Use product of root and conjugate	
			$b = 25$	A1	Obtain correct answer	
				[4]		
			OR			
				M1	Substitute $4+3i$ or conjugate into equation	
				M1	Equate real and imaginary parts	
			$a = -8$	A1	Obtain correct answer	
			$b = 25$	A1	Obtain correct answer	
4				M1	Express as sum of 3 series	
				M1	Use standard series results, at least 1 correct	
				A1	Two terms correct	
			$\frac{1}{2}n(n+1)(2n+1) - \frac{3}{2}n(n+1) + 2n$	A1	Third term correct	
				M1	Obtain factor of n	
			$n(n^2+1)$	A2	Obtain correct answer c.a.o.	
					Allow A1 for $\frac{1}{2(2n^2+2)}$	
				[7]		
5				B1	Verify result true when $n = 1$	
				M1*	Add next term in series	
				DepM1	Attempt to obtain 3^{k+1} correctly	
				A1	Show sufficient working to justify correct expression	
				B1	Clear statements of Induction processes, but 1st 4 marks must all be earned.	
				[5]		

Question			Answer	Marks	Guidance	
6	(i)		$5u^2 + 11u + 8 = 0$	M1 M1 A1 **[3]**	Attempt to clear fractions Attempt to expand and simplify to a quadratic Obtain correct answer, must be an equation	
6	(ii)		EITHER $u = \frac{1}{x} - 1$ $\frac{8}{5}$ OR $\frac{1}{\alpha\beta} - \frac{\alpha+\beta}{\alpha\beta} + 1$ $\frac{8}{5}$	 B1 M1 A1 FT **[3]** B1 M1 A1	 State or imply by using roots of new quadratic Use their c/a Obtain correct answer Express in terms of $\alpha + \beta$ and $\alpha\beta$ Use values $-\frac{1}{2}$ and $\frac{5}{2}$ correctly Obtain correct answer	 Must be values from original equation

Question			Answer	Marks	Guidance	
7	(i)			B1B1 B1ft B1ft B1B1 **[6]**	Circle, centre (3 , 4) Touching x-axis, ft for $(3, -4)$ etc as centre Crossing y-axis twice Horizontal line, y intercept 4	
7	(ii)		$-1 + 4i \quad 7 + 4i$	B1B1 **[2]**	State correct answers	
7	(iii)			B1ft B1 **[2]**	Inside circle or above line Completely correct diagram	
8	(i)			B1 **[1]**	Show **given** answer correctly	
8	(ii)		$1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using (i) Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms correct Show terms cancelling Obtain correct answer, must be in terms of n	
8	(iii)		$\frac{3}{2}$ $N = 4$	B1ft B1 M1 A1 **[4]**	State or use correct sum to infinity Their sum to infinity – their (ii) = $\frac{11}{30}$ Attempt to solve correct equation Obtain only $N = 4$	

Question			Answer	Marks	Guidance	
9	(i)			B1* depB1 **[2]**	Shear eg image of (0 , 1) is (2 , 1) or parallel to the *x*-axis	
9	(ii)		Either $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ **[5]** Or $\mathbf{Z} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} a & 2a+b \\ c & 2c+d \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	B1 B1 B1 M1 A1 B1 B1 B1 M1 A1	State **Z** = **YX** Obtain **Y** = **ZX**$^{-1}$ State or use correct inverse Matrix multiplication, 2 elements correct Obtain completely correct simplified exact matrix Correct order for matrix multiplication Obtain 2correct elements Obtain other 2 correct elements Equate elements, 2 correct Obtain completely correct simplified exact matrix	
9	(iii)			B1* depB1 **[2]**	Rotation 60° clockwise	

Question			Answer	Marks	Guidance	
10	(i)		$a^3 - 4a$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer	
10	(ii)	(a)		B1 **[1]**	det **D = 15** so unique sol'n or solve to find correct solution (-2/5, 1, 4/5)	**SC** B1 once if unique solution following their incorrect det **D** non zero
10	(ii)	(b)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 2$ Explain inconsistency with correct working	
10	(ii)	(c)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 0$ Explain consistency with correct working	